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Stabilization of Nonlinear Continuous–Discrete Dynamic Systems with a Constant Sampling Step

S. V. Akmanova

Nosov Magnitogorsk State Technical University, Magnitogorsk, Russia
e-mail: svet.akm_74@mail.ru

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Abstract—This paper considers nonlinear continuous–discrete (hybrid) systems containing two subsystems of differential and difference equations, respectively, and one-dimensional (scalar) or multidimensional (vector) control. The transition from a nonlinear hybrid system with a constant sampling step $h > 0$ to an equivalent, in a natural sense, nonlinear discrete dynamic system is presented. Sufficient conditions are established, first, for reducing the first approximation systems of nonlinear discrete systems to the Brunovský canonical form and, second, for stabilizing such systems and nonlinear hybrid systems with control of different dimensions. Algorithms for constructing stabilizing control laws for nonlinear hybrid systems are developed. Numerical examples are provided to illustrate the effectiveness of this approach to stabilizing nonlinear hybrid dynamic systems.

Keywords: continuous–discrete system, discrete system, hybrid system, controlled system, equilibrium, stabilizable system, Brunovský canonical form

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1. INTRODUCTION

Stabilization of dynamic systems is one of the most important problems in control theory, which is due to the demands of control practice and open (unsolved) scientific issues in this area [1–4]. The solution of this problem will ensure stable operating modes of dynamic systems and contribute to solution of control problems for these systems.

Hybrid dynamic systems serve as mathematical models of real mechanical, technical, technological, and other processes of a heterogeneous nature that cannot be described only by differential equations. For example, control of aircraft and electric trains can be modeled only using a discrete thrust regulator [5, 6]. Therefore, the main feature of such hybrid (continuous–discrete) dynamic control systems is their adequacy to modeled objects [7], which, as a rule, have nonlinear operation processes. Hence, the corresponding hybrid dynamic systems are nonlinear continuous–discrete systems.

Among hybrid systems, there is a large class of systems stabilized by switching at appropriate time instants [8]. However, the issues of stabilizing real dynamic control systems are inseparably connected with their controllability issues.

Methods for stabilizing controlled dynamic systems have been created and developed for over 150 years [4]. Much experience has been gained in stabilization of continuous systems and discrete systems; for example, see [9–16]. There are R&D results in the field of stabilizing linear hybrid [3, 7, 17] and nonlinear hybrid systems described by differential equations with different nonlinearities and a discrete state- or output-feedback controller [18–22]. However, the issues of stabilizing nonlinear hybrid systems with operation processes described by differential and difference equations and states containing both continuous and discrete components have not been sufficiently studied so far.

This paper presents a general approach to stabilizing such nonlinear continuous–discrete (hybrid) systems with one-dimensional (scalar) or multidimensional (vector) control and a constant sampling step. The approach is based on the transition from a given nonlinear hybrid system to an equivalent, in the natural sense, nonlinear discrete dynamic system.

The novelty of this study is as follows:

1) The concept of the Brunovský canonical form is introduced for the first approximation system of a nonlinear discrete dynamic system with scalar control.

2) Sufficient conditions are established for reducing the first approximation system of an equivalent nonlinear discrete system with scalar (vector) control to the Brunovský canonical form (i.e., to a set of independent subsystems each having the Brunovský canonical form).

3) Algorithms for reducing the first approximation systems of equivalent nonlinear discrete systems (with both scalar and vector control) to the Brunovski canonical form are developed and demonstrated by examples.

4) Sufficient conditions for stabilizing the first approximation systems of equivalent nonlinear discrete dynamic control systems (both with scalar and vector control) are established.

5) Sufficient conditions for stabilizing nonlinear hybrid dynamic control systems with control of different dimensions are established.

6) Algorithms for constructing stabilizing control laws for nonlinear hybrid systems with control of different dimensions are developed and demonstrated by examples.

2. PROBLEM STATEMENT

Consider a nonlinear continuous–discrete control system of the form

$$\begin{cases} x'(t) = f(x(t), y(t_k)), \quad t_k \leqslant t < t_{k+1} \\ y(t_{k+1}) = g(x(t_{k+1}), y(t_k), u(t_k)), \quad k = 0, 1, 2, \dots \end{cases} \tag{1}$$

with initial conditions

$$x(t_0) = x_0, \quad y(t_0) = y_0, \tag{2}$$

where $x \in R^n$ and $y \in R^m$ are the state vectors of system (1) characterizing the behavior of its continuous and discrete parts, respectively; $u \in R^q$ is the control vector (input) of system (1); the time instants t_k define a uniform grid on R with a constant step $h > 0$, i.e., $t_{k+1} - t_k = h > 0$, $k = 0, 1, 2, \dots$ and $t_k = kh$; the functions $f(x, y)$ and $g(x, y, u)$ are continuously differentiable in the aggregate of variables.

Assume that without control ($u = 0$),

$$f(0, 0) = 0, \quad g(0, 0, 0) = 0. \tag{3}$$

In other words, system (1) with $u = 0$ has the trivial equilibrium $x = 0$, $y = 0$.

System (1), (2) with a chosen control law $u = u(t_k)$, $k = 0, 1, 2, \dots$, operates in accordance with the following standard scheme:

1) Initial conditions $z_0 = (x_0, y_0)$ are specified.
2) The solution $x = \varphi_0(t)$ of the Cauchy problem $x' = f(x, y_0)$, $x(t_0) = x_0$, is found, and the vectors $x_1 = \varphi_0(t_1)$ and $y_1 = g(x_1, y_0, u_0)$, where $u_0 = u(t_0)$, are constructed.
3) The solution $x = \varphi_1(t)$ of the Cauchy problem $x' = f(x, y_1)$, $x(t_1) = x_1$, is found, and the vectors $x_2 = \varphi_1(t_2)$ and $y_2 = g(x_2, y_1, u_1)$, where $u_1 = u(t_1)$, are constructed. And so on.

Hence, the solution of system (1) with the initial point $z_0 = (x_0, y_0)$ and a chosen control law $u = u(t_k)$, $k = 0, 1, 2, \ldots$, is the function

$$z(t) = (x(t), y(t)) = \begin{cases} (\varphi_0(t), y_0), & t_0 \leqslant t < t_1 \\ (\varphi_1(t), y_1), & t_1 \leqslant t < t_2 \\ (\varphi_2(t), y_2), & t_2 \leqslant t < t_3 \\ \qquad \vdots \end{cases} \tag{4}$$

The components of the solution (4) have the following features: the function $x(t)$ is continuous for all $t \geqslant 0$, continuously differentiable on any interval (t_k, t_{k+1}), but not necessarily differentiable at time instants $t = t_k$, where $k = 0, 1, 2, \ldots$; the function $y(t)$ is piecewise constant and changes its values at time instants $t = t_k$, $k = 0, 1, 2, \ldots$.

The main problem of this paper is to establish sufficient conditions for stabilizing systems (1) and to design stabilizing control laws for such systems. The solution of this problem is based on the transition from the nonlinear hybrid system (1) to an equivalent nonlinear discrete dynamic system.

3. TRANSITION TO THE DISCRETE SYSTEM. CONTROLLABILITY AND STABILIZABILITY OF SYSTEMS

Due to the relations (3), the functions $f(x, y)$ and $g(x, y, u)$ can be represented as

$$f(x, y) = A_1 x + B_1 y + a(x, y), \quad g(x, y, u) = A_2 x + B_2 y + Cu + b(x, y, u),$$

where $A_1 = f'_x(0, 0)$, $B_1 = f'_y(0, 0)$, $A_2 = g'_x(0, 0, 0)$, $B_2 = g'_y(0, 0, 0)$, and $C = g'_u(0, 0, 0)$ are matrices of compatible dimensions and the smooth nonlinearities $a(x, y)$ and $b(x, y, u)$ satisfy the relations

$$a(x, y) = o(\|x\| + \|y\|) \quad \text{as} \quad \|x\| + \|y\| \to 0,$$
$$b(x, y, u) = o(\|x\| + \|y\| + \|u\|) \quad \text{as} \quad \|x\| + \|y\| + \|u\| \to 0.$$

By denoting $x(t_k) = x_k$, $y(t_k) = y_k$, and $u(t_k) = u_k$, we write system (1) as the equivalent hybrid system

$$\begin{cases} x'(t) = A_1 x(t) + B_1 y_k + a(x(t), y_k), & t_k \leqslant t < t_{k+1} \\ y_{k+1} = A_2 x_{k+1} + B_2 y_k + Cu_k + b(x_{k+1}, y_k, u_k), & k = 0, 1, 2, \ldots. \end{cases} \tag{5}$$

Let $\det A_1 \neq 0$; by applying the shift operator along the trajectories of system $x' = f(x, y)$ over the time from $t = 0$ to $t = h > 0$ (see [23]), we perform the transition from system (5) to a discrete system of the form

$$\begin{cases} x_{k+1} = e^{A_1 h} x_k + A_1^{-1}(e^{A_1 h} - I) B_1 y_k + \varepsilon(x_k, y_k; h) \\ y_{k+1} = A_2 e^{A_1 h} x_k + (A_2 A_1^{-1}(e^{A_1 h} - I) B_1 + B_2) y_k + Cu_k + \delta(x_k, y_k, u_k; h), \end{cases} \tag{6}$$

where

$$\varepsilon(x_k, y_k; h) = e^{(t_k + h)A_1} \int\limits_{t_k}^{t_k + h} e^{-sA_1} a(p(s, x_k, y_k), y_k) ds,$$

$x = p(t, x_k, y_k)$ is the solution of the Cauchy problem

$$x' = A_1 x + B_1 y_k + a(x, y_k), \quad x(t_k) = x_k\,;$$
$$\delta(x_k, y_k, u_k; h) = A_2 \varepsilon(x_k, y_k; h) + b(x_{k+1}, y_k, u_k). \tag{7}$$

Note that the case of a singular matrix A_1 can also be considered, but it will lead to substantially more complicated formulas.

The discrete system (6) can be written in the compact form

$$z_{k+1} = A(h)z_k + Bu_k + \xi(z_k, u_k, h), \quad k = 0, 1, 2, \dots, \tag{8}$$

where

$$z_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix}, \quad B = \begin{bmatrix} O \\ C \end{bmatrix}, \quad \xi(z_k, u_k, h) = \begin{bmatrix} \varepsilon(x_k, y_k; h) \\ \delta(x_k, y_k, u_k; h) \end{bmatrix},$$

$A(h)$ is a block matrix of order $(n+m)$:

$$A(h) = \begin{bmatrix} e^{A_1 h} & A_1^{-1}(e^{A_1 h} - I)B_1 \\ A_2 e^{A_1 h} & A_2 A_1^{-1}(e^{A_1 h} - I)B_1 + B_2 \end{bmatrix}. \tag{9}$$

System (6) with $u = 0$ (equivalently, system (8)) has the trivial equilibrium.

Based on [23] and condition (7), it can be established that

$$\varepsilon(x, y; h) = o(\|x\| + \|y\|) \quad \text{as} \quad \|x\| + \|y\| \to 0,$$
$$\delta(x, y, u; h) = o(\|x\| + \|y\| + \|u\|) \quad \text{as} \quad \|x\| + \|y\| + \|u\| \to 0.$$

Then the function $\xi(z, u, h)$ in system (8) satisfies the condition

$$\xi(z, u, h) = o(\|z\| + \|u\|) \quad \text{as} \quad \|z\| + \|u\| \to 0. \tag{10}$$

For a chosen control law $u = u(t_k)$, $k = 0, 1, 2, \dots$, the hybrid system (1) and the discrete system (6) are equivalent in the following sense [8]:

• If $(x(t), y_k)$ is the solution of system (1), (x_k, y_k) will be the solution of system (6), where $x_k = x(t_k)$.

• If (x_k, y_k) is the solution of system (6), $(x(t), y_k)$ will be the solution of system (1), where $x(t)$ is the solution of the Cauchy problem $x' = f(x, y_k), x(t_k) = x_k$.

Therefore, systems (1) and (8) are equivalent as well.

Following [24–26], we introduce several notions.

Definition 1. The hybrid system (1) is said to be controllable for $h = h_0 > 0$ if for any vectors $z^{(0)}$, $z^{(1)} \in R^{n+m}$, there exists a control law $u = u(t_k)$, $k = 0, 1, \dots, l-1$ $(l \in N)$, such that $z(t_l) = z^{(1)}$, $t_l = t_0 + lh_0$, for the solution $z = z(t)$ of system (1) with the initial condition $z(t_0) = z^{(0)}$.

Definition 2. The discrete system (8) is said to be controllable for $h = h_0 > 0$ if for any states $z^{(0)}$, $z^{(1)} \in R^{n+m}$, there exists a control law u_k, $k = 0, 1, \dots, l-1$ $(l \in N)$, such that $z_l = z^{(1)}$, $z_l = z(t_l) = z(t_0 + lh_0)$, for the solution z_k, $k = 0, 1, \dots, l$, of system (8) with the initial condition $z_0 = z^{(0)}$.

For some $h > 0$, the equilibrium of systems (1) and (8) may be unstable.

Definition 3. The hybrid system (1) is said to be stabilizable for $h = h_0 > 0$ if there exists a piecewise constant function $u = \varphi(t) \in R^q$, $t_k \leqslant t < t_{k+1}$, $k = 0, 1, 2, \dots$, such that system (1) with the control law $u = \varphi(t)$ has the asymptotically stable solution $x = 0$, $y = 0$ for $h = h_0 > 0$.

Definition 4. The discrete system (8) is said to be stabilizable for $h = h_0 > 0$ if there exists a control law $u_k = \Phi(z_k)$, $k = 0, 1, 2, \dots$, such that system (8) has the asymptotically stable solution $z = 0$ for $h = h_0 > 0$.

Direct comparison of Definitions 1 and 2 yields the following result.

Theorem 1. *The hybrid system* (1) *is controllable for* $h = h_0 > 0$ *if and only if the discrete system* (8) *is controllable for* $h = h_0 > 0$.

The proof of this theorem, as well as those of other important results of the paper, is postponed to the Appendix.

The linear discrete system

$$z_{k+1} = A(h)z_k + Bu_k, \quad k = 0, 1, 2, \dots, \tag{11}$$

is the first approximation system of the nonlinear discrete system (8).

Now we proceed to the issues of stabilizing system (1) considering its control peculiarities.

4. STABILIZATION OF THE HYBRID SYSTEM (1) WITH SCALAR CONTROL

Let $u \in R^1$ in system (1); then the first approximation system of the corresponding discrete system (8) takes the form

$$z_{k+1} = A(h)z_k + bu_k, \quad k = 0, 1, 2, \dots, \tag{12}$$

where $z \in R^{n+m}$, $A(h)$ is the matrix (9), b is a matrix of dimensions $(n + m) \times 1$, and $u \in R^1$.

Assume that for $h = h_0 > 0$, system (12) satisfies the complete reachability condition, i.e.,

$$\operatorname{rank}\,[b, A(h_0)b, A^2(h_0)b, \dots, A^{n+m-1}(h_0)b] = n + m. \tag{13}$$

which implies the controllability of system (12) for $h = h_0 > 0$.

Note that condition (13) can be relaxed, e.g., by imposing the complete controllability condition on system (12) for $h = h_0 > 0$; for details, see [11, pp. 268–269].

We have the following result.

Theorem 2. *Assume that the linear discrete system* (12) *satisfies condition* (13). *Then there exists a transformation*

$$z_k^* = Sz_k, \quad u_k^* = \alpha z_k + u_k, \tag{14}$$

where $\det S \neq 0$ *and* α *is a matrix of dimensions* $1 \times (n + m)$, *reducing this system to*

$$\begin{cases} z^*_{k+1,1} = z^*_{k,2} \\ z^*_{k+1,2} = z^*_{k,3} \\ \vdots \\ z^*_{k+1,n+m-1} = z^*_{k,n+m} \\ z^*_{k+1,n+m} = u^*_k, \end{cases} \tag{15}$$

where $z^*_{k,i}$ *denotes the ith component of the vector* z^*_k.

System (15) will be called the *Brunovský canonical form of system* (12) *for* $h = h_0 > 0$.

Theorem 3. *Assume that for* $h = h_0 > 0$, *system* (12) *satisfies condition* (13). *Then it is stabilizable for* $h = h_0 > 0$.

From the proof of Theorem 3 (see the Appendix) and equalities (14) we arrive at the following fact.

Corollary 1. *A control law* $u_k = S^* z_k$, $k = 0, 1, 2, \dots,$ *where* $S^* = pS - \alpha$, $u^*_k = pz^*_k$, *stabilizes the linear discrete system* (12), *and all eigenvalues of the matrix* $(A(h_0) + bS^*)$ *are smaller than 1 by their magnitude.*

Theorem 4. *If the first approximation system* (12) *of the corresponding nonlinear discrete system* (8) *satisfies condition* (13), *the corresponding hybrid system* (1) *will be stabilizable for* $h = h_0 > 0$.

Example 1. It is required to construct a stabilizing control law for a hybrid system of the form

$$\begin{cases} x'(t) = -x(t) + y_{k,1} + a(x(t), y_k), \quad t_k \leqslant t < t_{k+1} \\ y_{k+1,1} = x(t_{k+1}) + y_{k,2} - u_k + b_1(x(t_{k+1}), y_k, u_k) \\ y_{k+1,2} = -x(t_{k+1}) + y_{k,1} + 0.5u_k + b_2(x(t_{k+1}), y_k, u_k), \quad k = 0, 1, 2, \ldots, \end{cases} \tag{16}$$

where $x \in R^1$, $y \in R^2$, $u \in R^1$, $a(x, y)$, $b_1(x, y, u)$, and $b_2(x, y, u)$ are smooth nonlinearities in accordance with system (5), by reducing the corresponding discrete first approximation system to the Brunovský canonical form.

For a nonlinear discrete system equivalent to (16), the first approximation system is a discrete system of the form (12) with the matrices

$$A(h) = \begin{bmatrix} e^{-h} & 1 - e^{-h} & 0 \\ e^{-h} & 1 - e^{-h} & 1 \\ -e^{-h} & e^{-h} & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -1 \\ 0.5 \end{bmatrix}.$$

It can be proved that this discrete system with all $h:\ 0 < h \neq ln\, 4$ is controllable.

Letting $h_0 = ln2$ yields $A(h_0) = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 1 \\ -0.5 & 0.5 & 0 \end{bmatrix}$; note that the solution $x = 0$, $y = 0$ of system (16) with $u = 0$ is stable (but not asymptotically stable). We compile the matrix $F(h_0) = [b, A(h_0)b, A^2(h_0)b]$:

$$F(h_0) = \begin{bmatrix} 0 & -0.5 & -0.25 \\ -1 & 0 & -0.75 \\ 0.5 & -0.5 & 0.25 \end{bmatrix}; \quad \text{then } F^{-1}(h_0) = \begin{bmatrix} 6 & -4 & -6 \\ 2 & -2 & -4 \\ -8 & 4 & 8 \end{bmatrix},$$

$$f(h_0) = [-8 \;\; 4 \;\; 8], \quad S = \begin{bmatrix} f(h_0) \\ f(h_0)A(h_0) \\ f(h_0)A^2(h_0) \end{bmatrix} = \begin{bmatrix} -8 & 4 & 8 \\ -6 & 2 & 4 \\ -4 & 0 & 2 \end{bmatrix}.$$

Then $z_k^* = Sz_k$, which corresponds to the system

$$\begin{cases} z_{k,1}^* = -8x_k + 4y_{k,1} + 8y_{k,2} \\ z_{k,2}^* = -6x_k + 2y_{k,1} + 4y_{k,2} \\ z_{k,3}^* = -4x_k + 2y_{k,2}. \end{cases}$$

Consequently,

$$\begin{cases} z_{k+1,1}^* = -8x_{k+1} + 4y_{k+1,1} + 8y_{k+1,2} = -6x_k + 2y_{k,1} + 4y_{k,2} = z_{k,2}^* \\ z_{k+1,2}^* = -6x_{k+1} + 2y_{k+1,1} + 4y_{k+1,2} = -4x_k + 2y_{k,2} = z_{k,3}^* \\ z_{k+1,3}^* = -4x_{k+1} + 2y_{k+1,2} = -3x_k - y_{k,1} + u_k = u_k^*. \end{cases}$$

Thus, for $h_0 = ln2$, the Brunovský canonical form of the discrete first approximation system corresponding to system (16) is given by

$$\begin{cases} z_{k+1,1}^* = z_{k,2}^* \\ z_{k+1,2}^* = z_{k,3}^* \\ z_{k+1,3}^* = u_k^*, \end{cases}$$

where $u_k^* = -3x_k - y_{k,1} + u_k$. As is easily verified, $u_k^* = \alpha z_k + u_k$, where $\alpha = f(h_0)A^3(h_0)$.

Let us find a stabilizing control law for system (16).

We have $u_k = S^* z_k$, $S^* = pS - \alpha$, $u_k^* = p z_k^*$, where $p = [p_1 \;\; p_2 \;\; p_3]$, p_1, p_2, and p_3 are the coefficients of the characteristic equation of the last system of equations:

$$\lambda^3 - p_3\lambda^2 - p_2\lambda - p_1 = 0.$$

Since the choice of these coefficients is arbitrary, we assign them so that the roots λ_1, λ_2, and λ_3 of the characteristic equation lie inside the unit circle.

Taking $\lambda_{1,2,3} = \frac{1}{2}$ yields $\lambda^3 - \frac{3}{2}\lambda^2 + \frac{3}{4}\lambda - \frac{1}{8} = 0$, and $p = [\frac{1}{8} \;\; -\frac{3}{4} \;\; \frac{3}{2}]$, $\alpha = [-3 \;\; -1 \;\; 0]$; hence, $S^* = [\frac{1}{2} \;\; 0 \;\; 1]$, and $u_k = \frac{1}{2}x_k + y_{k,2}$ is the desired control law.

Indeed, for $h_0 = ln2$, substituting this control law into the discrete system (12) corresponding to system (16) gives the matrix $(A(h_0) + bS^*)$, for which all eigenvalues are equal to $\frac{1}{2}$ (smaller than 1 by their magnitude). Based on the proof of Theorem 4 (see the Appendix), we conclude that system (16) is stabilizable for $h_0 = ln2$ by the control law $u_k = \frac{1}{2}x_k + y_{k,2}$.

Consider an important special case of system (1) in which $n = m = q = 1$. In this case, system (1) contains scalar equations and is equivalent to the hybrid system

$$\begin{cases} x'(t) = a_1 x(t) + b_1 y_k + a(x(t), y_k), & t_k \leqslant t < t_{k+1} \\ y_{k+1} = a_2 x_{k+1} + b_2 y_k + c u_k + b(x_{k+1}, y_k, u_k), & k = 0, 1, 2, \ldots, \end{cases} \tag{17}$$

where $a_1 = f_x'(0,0)$, $b_1 = f_y'(0,0)$, $a_2 = g_x'(0,0,0)$, $b_2 = g_y'(0,0,0)$, and $c = g_u'(0,0,0)$ are numbers, and the smooth nonlinearities $a(x,y)$ and $b(x,y,u)$ satisfy the relations

$$a(x,y) = o(|x| + |y|) \quad \text{as} \quad |x| + |y| \to 0,$$
$$b(x,y,u) = o(|x| + |y| + |u|) \quad \text{as} \quad |x| + |y| + |u| \to 0\,.$$

The nonlinear discrete system equivalent to system (17) has the first approximation system (12) with

$$A(h) = \begin{bmatrix} e^{a_1 h} & \dfrac{b_1}{a_1}(e^{a_1 h} - 1) \\ a_2 e^{a_1 h} & \dfrac{a_2 b_1}{a_1}(e^{a_1 h} - 1) + b_2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ c \end{bmatrix}. \tag{18}$$

Then the following result is true.

Theorem 5. *If the hybrid system* (1) *contains scalar equations, and* $a_1 \neq 0$, $b_1 \neq 0$, *and* $c \neq 0$ *in the equivalent system* (17), *system* (1) *will be stabilizable for any* $h > 0$.

In this case, the stabilizing control law is found by the algorithm demonstrated in Example 1.

5. STABILIZATION OF SYSTEM (1) WITH VECTOR CONTROL

Let $n \geqslant 1$, $m > 1$, and $q > 1$ in the hybrid system (1), i.e., the discrete system (11) serves as the first approximation system for the equivalent discrete system (8). Assume that system (11) satisfies the complete reachability condition for $h = h_0 > 0$:

$$\operatorname{rank}[B, A(h_0)B, A^2(h_0)B, \ldots, A^{n+m-1}(h_0)B] = n + m. \tag{19}$$

Theorem 6. *If system* (11) *satisfies condition* (19), *there will exist a transformation*

$$z_k^* = S z_k, \quad u_{k,l}^* = \alpha_l z_k + u_{k,l}, \quad \det S \neq 0, \quad l = 1, \ldots, q, \tag{20}$$

reducing system (11) *to a set of* q *independent subsystems, each having the Brunovský canonical form.*

By the proof of this theorem (see the Appendix), the original system (11) decomposes into q independent subsystems of the form (15) whose dimensions are $s_1, \ldots, s_q$, with $s_1+s_2+\ldots+s_q = n+m$. Then we have the following result.

Theorem 7. *If the linear discrete system* (11) *satisfies condition* (19), *it will be stabilizable for* $h = h_0 > 0$.

The proof of Theorem 7 (see the Appendix) and equalities (20) lead to another fact.

Corollary 2. *The stabilizing control law for the discrete system* (11) *has the form*

$$u_k = S^* z_k,$$

where $u_{k,l} = s_l^* z_k$, $s_l^* = pS_l - \alpha_l$, $u_{k,l}^* = pz_{kl}^*$, S_l *is part of the matrix* S *corresponding to the* l*th subsystem* $(l = 1, \ldots, q)$, p *is a vector of dimension* s_l, z_{kl}^* *is the vector containing* s_l *components of the vector* z_k^* *corresponding to the* l*th subsystem, and* s_l^* *are the rows of the matrix* S^*. *In addition, all eigenvalues of the matrix* $(A(h_0) + BS^*)$ *are smaller than 1 by their magnitude.*

Theorem 8. *If the linear discrete system* (11) *satisfies condition* (19), *the corresponding nonlinear hybrid system* (1) *will be stabilizable for* $h = h_0 > 0$.

Example 2. It is required to construct a stabilizing control law for a hybrid system of the form

$$\begin{cases} x'(t) = -2x(t) + 2y_{k,1} - 2y_{k,2} + a(x(t), y_k), \quad t_k \leqslant t < t_{k+1} \\ y_{k+1,1} = -x_{k+1} + u_{k,1} - u_{k,2} + b_1(x_{k+1}, y_k, u_k) \\ y_{k+1,2} = x_{k+1} - y_{k,1} + y_{k,2} + u_{k,1} + 2u_{k,2} + b_2(x_{k+1}, y_k, u_k), \quad k = 0, 1, 2, \ldots, \end{cases}$$

where $x \in R^1$, $y \in R^2$, $u \in R^2$, $a(x, y)$, $b_1(x, y, u)$, and $b_2(x, y, u)$ are smooth nonlinearities in accordance with system (5), by reducing the corresponding discrete first approximation system to a set of independent subsystems written in the Brunovský canonical form.

For this system, the corresponding discrete system (11) has the matrices

$$A(h) = \begin{bmatrix} e^{-2h} & 1 - e^{-2h} & e^{-2h} - 1 \\ -e^{-2h} & e^{-2h} - 1 & 1 - e^{-2h} \\ e^{-2h} & -e^{-2h} & e^{-2h} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}.$$

Then we obtain

$$[B, A(h)B, A^2(h)B] = \begin{bmatrix} 0 & 0 & 0 & 3(e^{-2h} - 1) & 0 & (e^{-2h} - 1)(9e^{-2h} - 3) \\ 1 & -1 & 0 & 3(1 - e^{-2h}) & 0 & (e^{-2h} - 1)(3 - 9e^{-2h}) \\ 1 & 2 & 0 & 3e^{-2h} & 0 & e^{-2h}(9e^{-2h} - 6) \end{bmatrix},$$

and rank $[B, A(h)B, A^2(h)B] = 3$ for any $h > 0$. Hence, the corresponding linear discrete system (11) is controllable for any $h > 0$.

Let $h_0 = ln3$, and let b_1 and b_2 be the columns of the matrix B. Then

$$A_0 = A(h_0) = \begin{bmatrix} 1/9 & 8/9 & -8/9 \\ -1/9 & -8/9 & 8/9 \\ 1/9 & -1/9 & 1/9 \end{bmatrix}.$$

From the column sequence $(b_1, b_2, A_0 b_1, A_0 b_2, A_0^2 b_1, A_0^2 b_2)$ we compile the matrix F_0 of order 3 and rank 3. Note that the matrix $A_0 b_1$ is a zero column, so the columns $A_0 b_1$, and $A_0^2 b_1$ are excluded from consideration.

We have $A_0b_2 = [-8/3 \ \ 8/3 \ \ 1/3]^T$; therefore, $s_1 = 1$, $s_2 = 2$, and $F_0 = [b_1, b_2, A_0b_2]$, i.e.,

$$F_0 = \begin{bmatrix} 0 & 0 & -8/3 \\ 1 & -1 & 8/3 \\ 1 & 2 & 1/3 \end{bmatrix}, \quad F_0^{-1} = \begin{bmatrix} 17/24 & 2/3 & 1/3 \\ -7/24 & -1/3 & 1/3 \\ -3/8 & 0 & 0 \end{bmatrix},$$

$k = 1, 2,$ since $\operatorname{rank} B = 2$.

$k = 1: \ \sum_{j=1}^{k} s_j = s_1 = 1,$ hence, f_1 is the first row of the matrix F_0^{-1}.

$k = 2: \ \sum_{j=1}^{k} s_j = s_1 + s_2 = 3,$ hence, f_2 is the third row of the matrix F_0^{-1}.

$$S = \begin{bmatrix} f_1 \\ f_2 \\ f_2A_0 \end{bmatrix}, \text{ i.e., } S = \begin{bmatrix} 17/24 & 2/3 & 1/3 \\ -3/8 & 0 & 0 \\ -1/24 & -1/3 & 1/3 \end{bmatrix}.$$

$$z_k^* = Sz_k, \text{ then } \begin{cases} z_{k,1}^* = \dfrac{17}{24}x_k + \dfrac{2}{3}y_{k,1} + \dfrac{1}{3}y_{k,2} \\ z_{k,2}^* = -\dfrac{3}{8}x_k \\ z_{k,3}^* = -\dfrac{1}{24}x_k - \dfrac{1}{3}y_{k,1} + \dfrac{1}{3}y_{k,2}, \quad k = 0, 1, 2, \ldots, \end{cases}$$

$z_{k+1,1}^* = u_{k,1}^*,$ where $u_{k,1}^* = f_1A_0^{s_1}z_k + u_{k,1} = f_1A_0z_k + u_{k,1} = \dfrac{1}{24}x_k + u_{k,1};$

$z_{k+1,2}^* = z_{k,3}^*;$ $z_{k+1,3}^* = u_{k,2}^*,$ where $u_{k,2}^* = f_2A_0^{s_2}z_k + u_{k,2} = \dfrac{5}{72}x_k + \dfrac{2}{9}y_{k,1} - \dfrac{2}{9}y_{k,2} + u_{k,2}.$

Thus, for $h_0 = ln3$, the set of the Brunovský canonical forms for the linear discrete system corresponding to the original system has the form

$$\begin{cases} z_{k+1,1}^* = u_{k,1}^* \\ z_{k+1,2}^* = z_{k,3}^* \\ z_{k+1,3}^* = u_{k,2}^*. \end{cases}$$

Now we find a stabilizing control law for the corresponding linear discrete system for $h_0 = ln3$.

1) $u_{k,1}^* = pz_{k1}^*$, where $p = p_1$, $z_{k1}^* = z_{k,1}^*$, since $s_1 = 1$, $z_{k+1,1}^* = pz_{k,1}^*$, and then $\lambda = p$. Letting $\lambda = \frac{1}{3}$ yields $p = \frac{1}{3}$, $\alpha_1 = f_1A_0 = [\frac{1}{24} \ \ 0 \ \ 0]$, $S_1 = f_1$, and, hence, $s_1^* = pS_1 - \alpha_1 = [\frac{7}{36} \ \ \frac{2}{9} \ \ \frac{1}{9}]$, $u_{k,1} = s_1^*z_k$, i.e., $u_{k,1} = \frac{7}{36}x_k + \frac{2}{9}y_{k,1} + \frac{1}{9}y_{k,2}$;

2) $u_{k,2}^* = pz_{k2}^*$, where $p = [p_1 \ \ p_2]$, $z_{k2}^* = [z_{k,2}^* \ \ z_{k,3}^*]^T$, since $s_2 = 2$, $z_{k+1,3}^* = p_1z_{k,2}^* + p_2z_{k,3}^*$, and then $\lambda^2 - p_2\lambda - p_1 = 0$.

Letting $\lambda_{1,2} = \frac{1}{3}$ yields $\lambda^2 - \frac{2}{3}\lambda + \frac{1}{9} = 0$, and $p = [-\frac{1}{9} \ \ \frac{2}{3}]$;

$$\alpha_2 = f_2A_0^2 = [5/72 \ \ 2/9 \ \ -2/9], \quad S_2 = \begin{bmatrix} -3/8 & 0 & 0 \\ -1/24 & -1/3 & 1/3 \end{bmatrix},$$

$$s_2^* = pS_2 - \alpha_2 = [-1/18 \ \ -4/9 \ \ 4/9], \text{ i.e., } u_{k,2} = -\frac{1}{18}x_k - \frac{4}{9}y_{k,1} + \frac{4}{9}y_{k,2}.$$

Thus, the stabilizing control law has the form

$$u_k = \begin{bmatrix} \dfrac{7}{36}x_k + \dfrac{2}{9}y_{k,1} + \dfrac{1}{9}y_{k,2} \\ -\dfrac{1}{18}x_k - \dfrac{4}{9}y_{k,1} + \dfrac{4}{9}y_{k,2} \end{bmatrix}.$$

This law also stabilizes the original nonlinear hybrid system because

$$S^* = \begin{bmatrix} 7/36 & 2/9 & 1/9 \\ -1/18 & -4/9 & 4/9 \end{bmatrix}, \quad A_0 + BS^* = \begin{bmatrix} 1/9 & 8/9 & -8/9 \\ 5/36 & -2/9 & 5/9 \\ 7/36 & -7/9 & 10/9 \end{bmatrix},$$

and the matrix $(A_0 + BS^*)$ has the eigenvalues $\lambda_{1,2,3} = \frac{1}{3}$, i.e., $|\lambda_{1,2,3}| < 1$.

6. CONCLUSIONS

This paper has established a general sufficient condition for stabilizing nonlinear continuous–discrete systems of the form (1) with both one-dimensional (scalar) and multidimensional (vector) control. This condition is based on the complete reachability of the first approximation systems of the corresponding equivalent nonlinear discrete dynamic systems.

A general approach to stabilizing the nonlinear hybrid systems (1) with control of different dimensions has been presented, and stabilizing control laws for such systems have been designed. The approach can be used to ensure stable operation modes for real technical, mechanical, and other control systems with a heterogeneous structure as well as to construct admissible and optimal control laws for such systems.

APPENDIX

Proof of Theorem 1. The desired result follows from the equivalence of systems (1) and (8), which considers the relation of their solutions, and from the definitions of controllable systems.

The proof of Theorem 1 is complete.

Proof of Theorem 2. Since the linear discrete system (12) satisfies condition (13), the matrix

$$F(h_0) = \left[b,\ A(h_0)b,\ A^2(h_0)b,\ \ldots,\ A^{n+m-1}(h_0)b\right]$$

is nonsingular, i.e., there exists the inverse $F^{-1}(h_0)$.

Let the vector $f(h_0) \in R^{n+m}$ be composed of the elements of the last row of the matrix $F^{-1}(h_0)$ [27]. In view of the definition of an inverse, we obtain

$$f(h_0)b = f(h_0)A(h_0)b = \ldots = f(h_0)A^{n+m-2}(h_0)b = 0, \tag{A.1}$$

$$f(h_0)A^{n+m-1}(h_0)b = 1. \tag{A.2}$$

Letting

$$\begin{cases} z^*_{k,1} = f(h_0)z_k \\ z^*_{k,2} = f(h_0)A(h_0)z_k \\ z^*_{k,3} = f(h_0)A^2(h_0)z_k \\ \vdots \\ z^*_{k,n+m} = f(h_0)A^{n+m-1}(h_0)z_k \end{cases} \tag{A.3}$$

yields $z^*_k = Sz_k$, where $S = \begin{bmatrix} f(h_0) \\ f(h_0)A(h_0) \\ f(h_0)A^2(h_0) \\ \vdots \\ f(h_0)A^{n+m-1}(h_0) \end{bmatrix}$, and $\det S \neq 0$ since the discrete system (12) satisfies condition (13).

For $h = h_0 > 0$, system (12) takes the form

$$z_{k+1} = A(h_0)z_k + bu_k, \quad k = 0, 1, 2, \ldots;$$

due to (A.1) and (A.2), from equalities (A.3) it follows that

$$\begin{cases} z^*_{k+1,1} = f(h_0)z_{k+1} = z^*_{k,2} \\ z^*_{k+1,2} = f(h_0)A(h_0)z_{k+1} = z^*_{k,3} \\ \vdots \\ z^*_{k+1,n+m-1} = f(h_0)A^{n+m-2}(h_0)z_{k+1} = z^*_{k,n+m} \\ z^*_{k+1,n+m} = f(h_0)A^{n+m-1}(h_0)z_{k+1} = \alpha z_k + u_k, \end{cases} \tag{A.4}$$

where $\alpha = f(h_0)A^{n+m}(h_0)$.

Therefore, system (A.4) is given by (15) with $u^*_k = \alpha z_k + u_k$ and is equivalent to the discrete system (12) for $h = h_0 > 0$.

The proof of Theorem 2 is complete.

Proof of Theorem 3. As the discrete system (12) satisfies condition (13), by Theorem 2 it can be represented in the form (15).

Let $u^*_k = pz^*_k$, where $p = [p_1 \ \ p_2 \ \ \ldots \ \ p_{n+m}]$; then system (15) takes the form

$$\begin{cases} z^*_{k+1,1} = z^*_{k,2} \\ z^*_{k+1,2} = z^*_{k,3} \\ \vdots \\ z^*_{k+1,n+m-1} = z^*_{k,n+m} \\ z^*_{k+1,n+m} = p_1 z^*_{k,1} + p_2 z^*_{k,2} + \ldots + p_{n+m} z^*_{k,n+m}. \end{cases} \tag{A.5}$$

In addition, let $z^*_{k,1} = \lambda^0 = 1$, $z^*_{k,2} = z^*_{k+1,1} = \lambda$, $z^*_{k,3} = z^*_{k+1,2} = z^*_{k+2,1} = \lambda^2, \ldots, z^*_{k,n+m} = \lambda^{n+m-1}$, and $z^*_{k+1,n+m} = \lambda^{n+m}$. Then system (A.5) is reduced to the equation

$$\lambda^{n+m} - p_{n+m}\lambda^{n+m-1} - p_{n+m-1}\lambda^{n+m-2} - \ldots - p_2\lambda - p_1 = 0. \tag{A.6}$$

Equation (A.6) is characteristic for the matrix of system (A.5). Since the choice of the coefficients of equation (A.6) is arbitrary, we assign them so that $|\lambda_i| < 1$, $i = 1, \ldots, n+m$. Then system (15) with the control law $u^*_k = pz^*_k$ has the asymptotically stable solution $z^*_k = 0$. Hence, the discrete system (12) represented in the form (15) is stabilizable for $h = h_0 > 0$, and the proof of Theorem 3 is complete.

Proof of Theorem 4. The desired result is immediate from Corollary 1, the relation (10), and the sufficient condition for the asymptotic stability of the trivial equilibrium of the nonlinear continuous–discrete system (1) obtained in [23]. The proof of Theorem 4 is complete.

Proof of Theorem 5. It can shown that if $a_1 \neq 0$, $b_1 \neq 0$, and $c \neq 0$ in the nonlinear hybrid system (17), the corresponding discrete system (12) with the matrices (18) will satisfy condition (13). In this case, the desired result is true based on Theorem 4. The proof of Theorem 5 is complete.

Proof of Theorem 6. Without losing generality, we suppose that $\operatorname{rank} B = q$, $q > 1$. Let $b_1, \ldots, b_q$ be the columns of the matrix B, $A_0 = A(h_0)$, and the discrete system (11) satisfy condition (19).

From the column sequence (see [27])

$$(b_1, b_2, \ldots, b_q, A_0b_1, A_0b_2, \ldots, A_0b_q, A_0^2b_1, A_0^2b_2, \ldots, A_0^2b_q, \ldots, A_0^{n+m-1}b_1, \ldots, A_0^{n+m-1}b_q)$$

we compile the matrix $F_0 = F(h_0)$ of the same order and rank $(n+m)$:

$$F_0 = [b_1, A_0 b_1, A_0^2 b_1, \ldots, A_0^{s_1-1} b_1, b_2, A_0 b_2, A_0^2 b_2, \ldots,\\ A_0^{s_2-1} b_2, \ldots, b_q, A_0 b_q, A_0^2 b_q, \ldots, A_0^{s_q-1} b_q]. \tag{A.7}$$

For each column b_k, the matrix (A.7) should include all columns $b_k, A_0 b_k, A_0^2 b_k, \ldots, A_0^{s_k-1} b_k$, where $s_k = 1, \ldots, n+m$, $k = 1, \ldots, q$, with each column $A_0^j b_k$ $(j = 0, 1, \ldots, n+m-1)$ being included if it forms a linearly independent system with all preceding columns in this matrix. Otherwise, the columns $A_0^j b_k$, $A_0^{j+1} b_k, \ldots, A_0^{n+m-1} b_k$ are excluded from consideration.

Since $\operatorname{rank} F_0 = n+m$, there exists the inverse F_0^{-1}. Let f_k $(k = 1, \ldots, q)$ denote the row of the matrix F_0^{-1} numbered by $\sum\limits_{j=1}^{k} s_j$. According to the definition of an inverse, we obtain the conditions

$$f_k A_0^{j-1} b_i = 0, \quad (k \neq i) \vee (j \neq s_k), \tag{A.8}$$

$$f_k A_0^{s_k-1} b_k = 1. \tag{A.9}$$

Let

$$\begin{cases} z^*_{k,1} = f_1 z_k \\ z^*_{k,2} = f_1 A_0 z_k \\ \vdots \\ z^*_{k,s_1} = f_1 A_0^{s_1-1} z_k \\ z^*_{k,s_1+1} = f_2 z_k \\ z^*_{k,s_1+2} = f_2 A_0 z_k \\ \vdots \\ z^*_{k,s_1+s_2} = f_2 A_0^{s_2-1} z_k \\ \vdots \\ z^*_{k,s_1+s_2+\ldots+s_q} = f_q A_0^{s_q-1} z_k, \end{cases} \tag{A.10}$$

where $s_1 + s_2 + \ldots + s_q = n+m$. Then $z^*_k = S z_k$, where $S = \begin{bmatrix} f_1 \\ f_1 A_0 \\ \vdots \\ f_1 A_0^{s_1-1} \\ f_2 \\ f_2 A_0 \\ \vdots \\ f_2 A_0^{s_2-1} \\ \vdots \\ f_q \\ \vdots \\ f_q A_0^{s_q-1} \end{bmatrix}$, S is a matrix of order $(n+m)$ with $\det S \neq 0$ because the discrete system (11) satisfies condition (19).

Then $z^*_{k+1} = S z_{k+1}$; for $h = h_0 > 0$, the discrete system (11) takes the form

$$z_{k+1} = A_0 z_k + B u_k.$$

Considering (A.8)–(A.10), we obtain

$$\begin{cases} z^*_{k+1,1} = f_1A_0z_k + f_1Bu_k = f_1A_0z_k = z^*_{k,2} \\ z^*_{k+1,2} = f_1A_0^2z_k + f_1A_0Bu_k = f_1A_0^2z_k = z^*_{k,3} \\ \vdots \\ z^*_{k+1,s_1} = f_1A_0^{s_1}z_k + f_1A_0^{s_1-1}Bu_k = f_1A_0^{s_1}z_k + u_{k,1} = u^*_{k,1} \\ z^*_{k+1,s_1+1} = f_2A_0z_k + f_2Bu_k = z^*_{k,s_1+2} \\ \vdots \\ z^*_{k+1,s_1+s_2} = f_2A_0^{s_2}z_k + f_2A_0^{s_2-1}Bu_k = f_2A_0^{s_2}z_k + u_{k,2} = u^*_{k,2} \\ \vdots \\ z^*_{k+1,s_1+s_2+\ldots+s_q} = f_qA_0^{s_q}z_k + f_qA_0^{s_q-1}Bu_k = f_qA_0^{s_q}z_k + u_{k,q} = u^*_{k,q}\,. \end{cases}$$

Thus, for $h = h_0 > 0$, the transformation (20) reduces the discrete system (11) to a set of q independent subsystems in the Brunovský canonical form:

$$\begin{cases} z^*_{k+1,1} = z^*_{k,2} \\ z^*_{k+1,2} = z^*_{k,3} \\ \vdots \\ z^*_{k+1,s_1} = u^*_{k,1} \\ z^*_{k+1,s_1+1} = z^*_{k,s_1+2} \\ \vdots \\ z^*_{k+1,s_1+s_2} = u^*_{k,2} \\ \vdots \\ z^*_{k+1,s_1+s_2+\ldots+s_q-1} = z^*_{k,s_1+s_2+\ldots+s_q} \\ z^*_{k+1,n+m} = u^*_{k,q}\,. \end{cases} \tag{A.11}$$

The dimensions of these subsystems (the number of equations figuring in them) are $s_1, s_2, \ldots, s_q$, respectively. In addition, $u^*_{k,l} = \alpha_l z_k + u_{k,l}$ $(l = 1, \ldots, q)$ are the components of the control vector, and $\alpha_l = f_l A_0^{s_l}$. The discrete system (11) is equivalent to system (A.11) for $h = h_0 > 0$.

The proof of Theorem 6 is complete.

Proof of Theorem 7. By the proof of Theorem 3, each of the q subsystems in (A.11) has an asymptotically stable solution $z^* = 0$ under appropriate control laws. In this case, system (A.11) has the asymptotically stable trivial solution. Hence, the equivalent system (11) is stabilizable for $h = h_0 > 0$.

The proof of Theorem 7 is complete.

Proof of Theorem 8. It follows from Theorem 7 that the discrete system (11) is stabilizable for $h = h_0 > 0$ and, according to Corollary 2, all eigenvalues of the matrix $(A_0 + BS^*)$ are smaller than 1 by their magnitude. Then the desired result is immediate from the relation (10) and the sufficient condition for the asymptotic stability of the trivial equilibrium of the nonlinear continuous–discrete system (1) obtained in [23].

The proof of Theorem 8 is complete.

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